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Control of Electromagnetic Satellite Formations in Near-Earth Orbits

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Electromagnetic formation flying is a novel concept of controlling the relative degrees of freedom of a satellite formation without the expenditure of fuel by using high-temperature superconducting wires to create magnetic electromagnetic formation flying. Because of inherent nonlinearities and couplings, the dynamics and control problem associated with electromagnetic formation flying are difficult, especially for near-Earth operations. This paper presents the application of nonlinear adaptive control laws that enable formation maintenance and reconfiguration. An approach to control allocation as the solution of an optimization problem is also proposed. The accumulation of angular momentum in the presence of Earth's magnetic field is an issue with electromagnetic formation flying and ways of managing it by exploiting the nonlinearity of magnetic dipoles using polarity switching are presented as a solution. Closed-loop nonlinear simulation results are also presented to demonstrate the feasibility and importance of the control scheme described for electromagnetic formation flying for near-Earth operations.

Nomenclature

$\mathbf{B}_e(\mathbf{R})$	=	Earth's magnetic field vector at location $\mathbf{R}$
$\mathbf{B}_j$	=	magnetic field strength due to satellite j at the location of ith satellite
$\mathbf{F}^d$, $\boldsymbol{\tau}_d$	=	disturbance force and torque vectors
$\mathbf{F}^m_{ij}$, $\boldsymbol{\tau}^m_{ij}$	=	magnetic force and torque vectors acting on satellite i due to satellite j
${}^{\mathrm{FI}}\mathbf{F}$	=	vector $\mathbf{F}$ represented in coordinates in the in the Earth-centered inertial frame
$\mathbf{f}$	=	specific force vector
J_k	=	kth zonal harmonic of Earth ($J_2 = 0.001082$)
m_i	=	mass of the ith satellite
P_k	=	Legendre polynomials of the first kind
p_i	=	position vector of ith satellite in the frame orbital frame F^O, $[x_{0i} \quad y_{0i} \quad z_{0i}]^T = [p_{ix} \quad p_{iy} \quad p_{iz}]^T$
$\mathbf{q}$	=	generalized coordinates of the system, $[\mathbf{q}_0 \quad \mathbf{q}_1 \quad \mathbf{q}_2 \cdots \mathbf{q}_{N-1}]^T$
$\mathbf{q}_i$	=	generalized coordinates of the ith satellite
R_e	=	equatorial radius of Earth, 6.378×10^6 m
$\mathbf{R}_i$	=	position vector of the ith satellite, $[x_i^T, y_i^T, z_i^T]^T$
R_i	=	$\lvert\mathbf{R}_i\rvert = \sqrt{x_i^2 + y_i^2 + z_i^2}$
$\dot{\mathbf{R}}_i$	=	$\mathrm{d}\mathbf{R}_i/\mathrm{d}t$
$\mathbf{r}_{ij}$	=	vector position of jth satellite with respect to the ith satellite
η_m	=	magnetic longitude
λ_m	=	latitude with respect to geomagnetic equatorial plane
μ_e	=	gravitational constant of Earth, $398,600 \times 10^9$ m^3/s^2
$\boldsymbol{\mu}_i$	=	magnetic dipole strength generated by the coils of the ith satellite
μ_m	=	Earth's dipole strength, 8×10^{22} A-m^2
$\boldsymbol{\tau}^{\mathrm{RW}}$	=	reaction wheel torques, $[\tau_x^{\mathrm{RW}} \quad \tau_y^{\mathrm{RW}} \quad \tau_z^{\mathrm{RW}}]^T$
ϕ	=	angle between Earth north pole direction and $\mathbf{R}_i$
Ω_k	=	angular velocities of reaction wheel k.
$\boldsymbol{\omega}$	=	angular velocity of the satellite in Earth-centered inertial reference frame, $[\omega_1 \quad \omega_2 \quad \omega_3]^T$
$(\cdot)$	=	inner-product operator
$(\times)$	=	cross-product operator

I. Introduction

SATELLITE formation flying is an enabling technology for many space missions. There are a number of future planned missions that will take advantage of the distributed satellite architecture, made possible by flying a number of satellites in close formations, as opposed to a large monolithic structure. Usually, depending upon the mission scenario, there is a tight requirement to control the relative degrees of freedom (both translational and attitude) among formation satellites. There are many studies that have presented control schemes for controlling satellite formations using conventional thruster-based systems [1]. These schemes may require continuous expenditure of fuel to maintain formation geometry that can contaminate the sensitive sensors on board and mission lifetime also becomes dependent on the fuel available.

Different approaches to using field forces to control the relative motion of satellites have been proposed in the literature. Schaub [2] has presented electrostatic force approaches to maintain a formation, and Shoer and Peck [3] have considered flux pinning using HTS electromagnetic forces to achieve passively stable configurations. EMFF has advantages in terms of controllability, however, since any set of magnetic forces can be easily generated by controlling the magnitude of currents running through magnetic coils.

This paper addresses the novel concept of electromagnetic formation flying (EMFF) in which high-temperature superconducting (HTS) wire is used to create electromagnets that can be used to control all the relative translational degrees of freedom of the formation (for details, see [4,5]). Although EMFF can make the system's lifetime independent of fuel availability, the dynamics and control problem associated with formation flying become highly challenging, due to the nonlinear nature of the magnetic forces.

Previous work on EMFF has shown the potential applications and feasibility of this concept from a system perspective [6]. Designs based on mission efficiency metrics for case scenarios of interferometric systems estimate mass fractions and volume requirements that indicate advantages in the use of EMFF when compared to other propulsion systems, especially when long mission lifetimes are considered. Additionally, it can be shown that the relative degrees of freedom of a satellite formation in which each satellite is fully

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actuated (i.e., has three orthogonal coils and three orthogonal reaction wheels) is controllable (the center of mass of the system cannot be moved by using relative actuation such as electromagnetic force) [7].

In a previous paper [8] it was shown that a general N-satellite electromagnetic formation can be stabilized and a framework for generating optimal time trajectories was presented along with some results. EMFF has distinct advantages over conventional means of maintaining formations in low Earth orbits (LEO) and medium Earth orbits due to high-ΔV requirements [9]. Near-Earth operation of electromagnetic formations is complicated by the fact that Earth has a strong magnetic field that results in considerable unwanted magnetic torques, thus making the angular momentum management an issue. In the present paper a method to manage the angular momentum built up by the interaction with Earth's magnetic field is presented. This method exploits the inherent nonlinearity of the magnetic field interaction.

The work in this paper addresses the following issues: First, the description of the dynamic equations for a formation of electromagnetic satellites includes the Earth's magnetic field, which allows posing a control approach based on adaptive control theory and verifying its performance through simulation. Second, a method for control allocation for EMFF systems that generates the desired forces on each vehicle based on the solution to an optimization problem leads to a more adequate implementation. Third, a technique to manage the angular momentum built up by the interaction with Earth's magnetic field shows how the proposed control approach is fit for this type of application by adapting to estimated model uncertainties.

In the second section of the paper, the nonlinear translational dynamics of an arbitrary N-satellite electromagnetic formation are described. In the third section, the attitude dynamics are derived. In the fourth section, nonlinear adaptive control laws are presented, for both translational and attitude control, that take into account the uncertainty in the actuation action and fluctuations in the Earth's magnetic field and the methods that can be used to solve for the dipole configuration to allocate the currents required to achieve the desired forces. In the fifth section, we address the problem of angular momentum buildup, and in the last section the results of a nonlinear simulation of the controlled system are presented.

II. Translational Dynamics

In this section, nonlinear equations of translational motion for general electromagnetic satellite formations will be derived in the ECI (Earth-centered inertial) frame. The nonlinear dynamics derived here are useful for simulating a general satellite formation orbiting Earth and will be used in the simulations presented in Sec. VI of this paper. This last version of the equations of motion is conveniently used in a later section for the formulation of the control laws.

A. Preliminaries

The ECI frame is defined with its origin at the center of Earth, its x axis points toward vernal equinox, z axis toward celestial north pole, and y axis completes a right-handed axis system. This axis system is not fixed to Earth (i.e., this frame does not rotate with Earth), although it moves as Earth orbits around the sun. For the purposes of this paper, this frame can be assumed to be an inertial frame.

The dynamics of relative positions will be developed in an orbital frame F^{RO}. This orbital frame is defined with its origin attached to the center of mass of the formation, its y axis aligned with the position vector $\mathbf{R}_{RO}$ representing the position of the formation center of mass in the ECI frame. Its z axis points toward the orbital plane normal and the x axis completes the right-hand system (see Fig. 1). A body frame F^{Bi} attached to the body of satellite i with its origin at the center of mass of the satellite is also defined to describe the orientation of each satellite in the inertial space. The location of satellite i is defined by the vector $\mathbf{R}_i$ that points from the Earth origin to the center of mass of satellite i.

The dynamics are derived for a general N-satellite electromagnetic formation such that satellites are enumerated from 0 to $N-1$, with satellite 0 designated as the leader satellite.

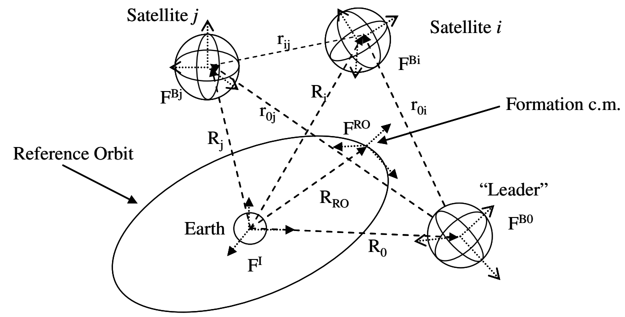

Fig. 1 Geometry of different reference frames.

B. Derivation of Translational Equations of Motion

By approximating the coils on each satellite to a steerable dipole, the magnetic field due to the jth satellite can be written as [9]

$$\mathbf{B}_j(\mathbf{r}_{ij}) = \frac{\mu_0}{4\pi}\left(\frac{3\boldsymbol{\mu}_j \cdot \mathbf{r}_{ij}}{r_{ij}^5}\mathbf{r}_{ij} - \frac{\boldsymbol{\mu}_j}{r_{ij}^3}\right) \tag{1}$$

and the force between satellites can be written as

$$\mathbf{F}_{ij}^m(\mathbf{q}_i, \mathbf{q}_j, \boldsymbol{\mu}_i, \boldsymbol{\mu}_j) = \frac{3\mu_0}{4\pi}\left(-\frac{\boldsymbol{\mu}_i \cdot \boldsymbol{\mu}_j}{r_{ij}^5}\mathbf{r}_{ij} - \frac{\boldsymbol{\mu}_i \cdot \mathbf{r}_{ij}}{r_{ij}^5}\boldsymbol{\mu}_j - \frac{\boldsymbol{\mu}_j \cdot \mathbf{r}_{ij}}{r_{ij}^5}\boldsymbol{\mu}_i + 5\frac{(\boldsymbol{\mu}_i \cdot \mathbf{r}_{ij})(\boldsymbol{\mu}_j \cdot \mathbf{r}_{ij})}{r_{ij}^7}\mathbf{r}_{ij}\right) \tag{2}$$

Note that Eq. (2) gives the force on dipole i (present on satellite i) due to dipole j (located on satellite j) and it depends on the distance between the two dipoles and the orientation of both dipoles in the inertial space. It is the dependence on the orientation of the dipoles that gives rise to the complexity of the expression for the force, since the orientation of a dipole obviously depends on its orientation with respect to the body frame of the satellite; it also depends on the orientation of the body axes in the inertial space.

In a similar fashion, the gradient of the Earth's magnetic field strength gives the force on the ith satellite due to the Earth's magnetic field. The magnetic field of Earth can be described as a combination of main field, which is due to the core of the planet, and an external component (that can be as much as 10% of the main field), which is due to sources such as ionosphere and solar wind effects on the Earth's magnetosphere [10].

The main field of Earth can be modeled as a dipole located at the center of the Earth. The orientation of this dipole varies slowly over time and the current magnetic north pole location is at 71.78°W and 79.74°N (2005 estimate). By defining a geomagnetic reference frame F^m, the magnetic field at the location of satellite i can be expressed as [10]

$${}^{Fm}\mathbf{B}_e(\mathbf{R}_i) = -\frac{\mu_0\mu_m}{R_i^3}\begin{bmatrix} 3\sin\lambda_m\cos\lambda_m\cos\eta_m \\ 3\sin\lambda_m\cos\lambda_m\sin\eta_m \\ 3\sin^2\lambda_m - 1 \end{bmatrix} \tag{3}$$

Another aspect that can be seen from Eq. (3) is that the magnitude of the gradient of the Earth's magnetic field scales as

$$|\nabla B_e| \sim \frac{\mu_m}{R_i^4} \tag{4}$$

and has a value of roughly 10^{-10} Wb/m^3 (for a 500 km orbit), which would result in a force less than 1 μN on a dipole having strength of 10^4 A-m^2. Since interdipole forces in an electromagnetic formation are generally of the order of 1 mN, one can safely ignore the magnetic force on the formation due to Earth's magnetic field and consider it as a disturbance force in the control design process. If this force is significant as compared to interdipole forces, then it can be explicitly taken into account.

It should be noted that the dipole model of the Earth's magnetic field is only an approximation of the main field component and it can have significant errors [10]. For a more realistic model of the Earth's magnetic field, existing geomagnetic models such International Geomagnetic Reference Field or World Magnetic Model can be used. These models predict only the Earth's main field components, however, and it is essential that any control scheme for EMFF accounts for the uncertainty present in the magnetic model by using online estimation or direct measurements.

C. Relative Translational Dynamics in a Circular Orbit

The formation translational control is based on the relative translational dynamics in the previous section. Taking into account only the most significant term of the gravity potential (i.e., $J_k = 0$), the translational dynamics of satellite i in the orbital reference frame F^o frame following a circular reference orbit are written as [7,11]

$$\ddot{x}_{ki} - 2\omega_0\dot{y}_{ki} - \omega_0^2 x_{ki} + \frac{\mu_e x_{ki}}{R_i^3} = f_{ix}^m - f_{kx}^m + f_{ix}^d - f_{kx}^d$$
$$\ddot{y}_{ki} + 2\omega_0\dot{x}_{ki} - \omega_0^2 y_{ki} + \frac{\mu_e(R_k + y_{ki})}{R_i^3} - \frac{\mu_e}{R_k^2} = f_{iy}^m - f_{ky}^m + f_{iy}^d - f_{ky}^d$$
$$\ddot{z}_{ki} + \frac{\mu_e z_{ki}}{R_i^3} = f_{iz}^m - f_{kz}^m + f_{iz}^d - f_{kz}^d \tag{5}$$

where the specific magnetic force components and specific disturbance force components are expressed in the orbital frame F^{R0}. The specific magnetic force acting on satellite i is defined as

$$\mathbf{f}_i^m = [f_{ix}^m \quad f_{iy}^m \quad f_{iz}^m]^T = \frac{1}{m_i}[F_{ix}^m \quad F_{iy}^m \quad F_{iz}^m]^T$$

and

$$\omega_0 = \sqrt{\frac{\mu_e}{R_{RO}^3}}$$

Note that the specific relative disturbance forces $\mathbf{f}_i^d - \mathbf{f}_k^d$ in Eq. (5) include all the unmodeled dynamics and perturbations such as differential J_2 and higher-order terms, differential solar pressure, differential drag, etc. The electromagnetic force provided by the coils needs to be strong enough to cancel these disturbances and at the same time provide additional force for trajectory-following if required. These equations will be used in Sec. IV for the derivation of positional control laws.

III. Attitude Dynamics

As discussed previously, a fully actuated satellite that has three orthogonal coils can control all the relative translational degrees of freedom. One side effect of applying any shear force using magnetic dipoles is that a torque also acts on the dipoles [7]; therefore, to control the attitude of a satellite and to counter this magnetic torque, each satellite in the formation needs to have angular momentum storage devices such as reaction wheels or control moment gyros. In this section, attitude dynamics of a satellite in electromagnetic formation will be presented in a simple form in order to highlight the control issues associated with attitude control. We will assume that each satellite has three orthogonal reaction wheels and will include their gyro-stiffening effect in the dynamic equations.

Consider Euler equations

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 + \omega_2 I_{\mathrm{RW3}}\Omega_3 - \omega_3 I_{\mathrm{RW2}}\Omega_2 = \tau_x^m + \tau_x^d - \tau_x^{\mathrm{RW}}$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 + \omega_3 I_{\mathrm{RW1}}\Omega_1 - \omega_1 I_{\mathrm{RW3}}\Omega_3 = \tau_y^m + \tau_y^d - \tau_y^{\mathrm{RW}}$$
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 + \omega_1 I_{\mathrm{RW2}}\Omega_2 - \omega_2 I_{\mathrm{RW1}}\Omega_1 = \tau_z^m + \tau_z^d - \tau_z^{\mathrm{RW}} \tag{6}$$

where

$$I = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$$

is the inertia matrix for the satellite;

$$I_{\mathrm{RW}} = \begin{bmatrix} I_{\mathrm{RW1}} & 0 & 0 \\ 0 & I_{\mathrm{RW2}} & 0 \\ 0 & 0 & I_{\mathrm{RW3}} \end{bmatrix}$$

is the inertia matrix for the reaction wheels; and $\boldsymbol{\tau}_{x,y,z}^m$, $\boldsymbol{\tau}_{x,y,z}^d$, and $\boldsymbol{\tau}_{x,y,z}^{\mathrm{RW}}$ are the components of the torques expressed in the satellite body frame (aligned with principle axes).

The magnetic torque term in Eq. (6) can be written as

$$\boldsymbol{\tau}_i^m = \sum_{j=0, j\neq i}^{N-1} \boldsymbol{\tau}_{ij}^m \tag{7}$$

and $\boldsymbol{\tau}_{ij}^m$ can be written as

$$\boldsymbol{\tau}_{ij}^m = \boldsymbol{\mu}_i \times \mathbf{B}_j(\mathbf{r}_{ij}) \tag{8}$$

Again, as done for the case of magnetic force, using the dipole approximation of the coils on each satellite, the magnetic torque on the ith satellite due to jth satellite can be written using Eq. (1) as

$$\boldsymbol{\tau}_{ij}^m = \frac{\mu_0 \boldsymbol{\mu}_i}{4\pi} \times \left(\frac{3\mathbf{r}_{ij}(\boldsymbol{\mu}_j \cdot \mathbf{r}_{ij})}{r_{ij}^5} - \frac{\boldsymbol{\mu}_j}{r_{ij}^3} \right) \tag{9}$$

Note that the torque depends upon the orientation of both the source and target dipoles.

A very important factor present in the disturbance torque term in Eq. (6) is the torque acting on each satellite due to the Earth's magnetic field. This torque can be written as

$$\boldsymbol{\tau}_i^e = \boldsymbol{\mu}_i \times \mathbf{B}_e(\mathbf{R}_i) \tag{10}$$

where $\mathbf{B}_e$ is Earth's magnetic field strength at the location of the satellite described in Eq. (3). As mentioned in Sec. III, the magnetic field of the Earth can be approximated by a dipole at the center of the Earth.

IV. Control Formulation

As discussed in previous sections, the dynamics of electromagnetic formation are not only highly nonlinear, they are coupled as well. Changing the dipole on one satellite affects actuation on all other satellites in the formation. Because of this coupled nature of dynamics, a hybrid control scheme is proposed in which translation control is implemented in a centralized fashion with a decentralized attitude control. For near-Earth operations of the electromagnetic formation, the control design is further complicated by the uncertainty present in the Earth's magnetic field model. Moreover, the magnetic force and torque models discussed so far are based on far-field models using dipole approximations that give accurate results only in the regions when the dipoles operate at distances much greater than the coil radii [9]. For near-field operations the error in the force and torque, as predicted by far-field models, can be as large as 10%, depending on the orientation of the dipoles. Given these factors, an adaptive control scheme is proposed in which these uncertainties in the models are estimated online by treating them as slowly varying parameters. In this section we consider the derivation of an adaptive controller formulation inspired in previous models presented for similar systems [12,13].

A. Centralized Adaptive Position Tracking Control for N-Satellite Electromagnetic Formation

As discussed above, due to the coupled nature of the dynamics, a centralized position tracking control scheme will be presented here. Consider a general N-satellite electromagnetic formation and let the satellites be numbered according to the index set $\mathfrak{I} = \{0, 1, \ldots, n\}$, where $n = N - 1$. Designating satellite 0 as the leader of the formation, the translational dynamics of the ith satellite in the formation, relative to the leader, are given by Eq. (6) and can be written in a compact form as follows:

$$\ddot{p}_i + c_i(\omega_0, \dot{p}_i) + g_i(R_0, p_i) = a_i^m + a_i^d \tag{11}$$

where $c_i(\omega_0, p_i) = [-2\omega_0 \dot{p}_{iy} \quad 2\omega_0 \dot{p}_{ix} \quad 0]^T$ is the Coriolis-like term,

$$g_i(R_0, p_i) = \begin{bmatrix} \frac{\mu_e p_{ix}}{R_i^3} - \omega_0^2 p_{ix} \\ \frac{\mu_e (R_0 + p_{iy})}{R_i^3} - \frac{\mu_e}{R_0^2} - \omega_0^2 p_{iy} \\ \frac{\mu_e p_{iz}}{R_i^3} \end{bmatrix}$$

is a gravity term,

$$a_i^m = \begin{bmatrix} f_{ix}^m - f_{0x}^m \\ f_{iy}^m - f_{0y}^m \\ f_{iz}^m - f_{0z}^m \end{bmatrix}$$

is the relative specific magnetic force on satellite i in frame F^O, and

$$a_i^d = \begin{bmatrix} f_{ix}^d - f_{0x}^d \\ f_{iy}^d - f_{0y}^d \\ f_{iz}^d - f_{0z}^d \end{bmatrix}$$

is the relative specific disturbance force on satellite i in frame F^O.

The relative specific magnetic force on satellite i, a_i^m, can be defined as a sum of far-field approximation and a near-field modifier term as follows:

$$a_i^m = \hat{a}_i^m + \begin{bmatrix} \hat{f}_{ix}^m - \hat{f}_{0x}^m & 0 & 0 \\ 0 & \hat{f}_{iy}^m - f_{0y}^m & 0 \\ 0 & 0 & \hat{f}_{iz}^m - \hat{f}_{0z}^m \end{bmatrix} \begin{bmatrix} \gamma_{ix} \\ \gamma_{iy} \\ \gamma_{iz} \end{bmatrix} \tag{12}$$

where

$$\hat{a}_i^m = \begin{bmatrix} \hat{f}_{ix}^m - \hat{f}_{0x}^m \\ \hat{f}_{iy}^m - \hat{f}_{0y}^m \\ \hat{f}_{iz}^m - \hat{f}_{0z}^m \end{bmatrix}$$

is the far-field approximation of the relative specific magnetic force on satellite i in frame F^O, and $\gamma_i = [\gamma_{ix} \quad \gamma_{iy} \quad \gamma_{iz}]^T$ is the unknown gain vector for relative magnetic force.

Note that in this formulation of the relative specific magnetic force, $\hat{a}_i^m$ is what is known from the model and γ_i is the unknown multiplication factor that adjusts the forces to actual level based on whether the satellite is in the near-field or far-field region. Since the motion of a satellite into and out of these regions is very slow as compared to the time constant of the dipoles, we can treat these factors as parameters that are slowly varying in time. In a similar fashion, treating the unknown disturbance forces as slowly varying parameters $d_i = [d_{ix} \quad d_{iy} \quad d_{iz}]^T$, the relative translational equations for satellite i can be written as

$$\ddot{p}_i + c_i(\omega_0, \dot{p}_i) + g_i(R_0, p_i) = \hat{a}_i^m + Y_i \theta_i \tag{13}$$

where

$$Y_i = \begin{bmatrix} \hat{f}_{ix}^m - \hat{f}_{0x}^m & 0 & 0 & 1 & 0 & 0 \\ 0 & \hat{f}_{iy}^m - f_{0y}^m & 0 & 0 & 1 & 0 \\ 0 & 0 & \hat{f}_{iz}^m - \hat{f}_{0z}^m & 0 & 0 & 1 \end{bmatrix}$$

and, $\theta_i = [\gamma_{ix} \quad \gamma_{iy} \quad \gamma_{iz} \quad d_{ix} \quad d_{iy} \quad d_{iz}]^T$ is the parameter vector for satellite i. Note that in this formulation other parameters such as mass, etc., if unknown, can also be incorporated [12,13].

By defining a relative position vector for the whole formation as $p = [p_1 \quad p_2 \quad \ldots \quad p_n]^T$, the relative dynamics of the whole formation can be written compactly as

$$\ddot{p} + C(\omega_0, \dot{p}) + G(R_0, p) = \hat{a}^m + Y\theta \tag{14}$$

where vectors C, etc., are obtained by stacking individual vectors c_i, etc., and Y is a block diagonal matrix of size $3n \times 6n$ that has Y_i at its diagonal.

Consider a trajectory-following problem in which each satellite is required to follow a predefined trajectory that is sufficiently slowly varying (i.e., time constant of the change in trajectory is much less than that of the system) and sufficiently smooth: i.e., $p_d(t) \in C^2$. Note that the desired trajectory vector $p_d(t) = [p_{d1}(t) \quad p_{d2}(t) \quad \cdots \quad p_{dn}(t)]^T$ defines trajectories for the satellites relative to the leader and does not include the trajectory for the leader itself, which is appropriate for EMFF, since with EMFF, the center of mass of the formation cannot be moved. In other words, $p_d(t)$ defines all the relative translational degrees of freedom of the formation that can be controlled. Let $\tilde{p}(t) = p(t) - p_d(t)$, $\dot{\tilde{p}}(t) = \dot{p}(t) - \dot{p}_d(t)$, and $\tilde{\theta} = \hat{\theta} - \theta$ be the position error, velocity error, and parameter error vectors, respectively. Define a composite error vector as follows:

$$s(t) = \dot{\tilde{p}}(t) + \lambda \tilde{p}(t) \tag{15}$$

where $\lambda > 0$ is a diagonal gain matrix. Rate of change of the composite error s can be written as

$$\dot{s}(t) = \ddot{p}(t) - \ddot{p}_r(t) \tag{16}$$

where

$$\ddot{p}_r(t) = \ddot{p}_d(t) + \lambda \dot{p}_d(t) - \lambda \dot{p}(t) \tag{17}$$

Consider a Lyapunov function as follows [12]:

$$V = \tfrac{1}{2} s^T s + \tfrac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{18}$$

where $\Gamma > 0$ is a diagonal gain matrix. The time derivative of the Lyapunov function is given as

$$\dot{V} = s^T \dot{s} + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \tag{19}$$

Using Eqs. (14) and (16), Eq. (19) can be written as

$$\dot{V} = s^T\{-C(\omega_0, \dot{p}) - G(R_0, p) + \hat{a}^m + Y\hat{\theta} - \ddot{p}_r(t)\} - s^T Y \tilde{\theta} + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \tag{20}$$

By choosing the control law with $K_p > 0$,

$$\hat{a}^m = C(\omega_0, \dot{p}) + G(R_0, p) - Y\hat{\theta} + \ddot{p}_r(t) - K_p s \tag{21}$$

and the adaptation law,

$$\dot{\hat{\theta}} = \Gamma Y^T s \tag{22}$$

the rate of change of Lyapunov function, Eq. (20) reduces to

$$\dot{V} = -s^T K_p s \tag{23}$$

and asymptotic stability follows from Barbalat's lemma [12].

The control law, Eq. (21), gives the desired specific force vector that can be used as input for a thruster-based system, but for EMFF the left-hand side of Eq. (21) is a complicated function of current position, orientation, and dipole strengths of each satellite in the formation given by Eq. (2). The control variables that we can actually change consist of the dipoles of individual satellites. To emphasize this dependence, the control law can be written as

$$\hat{a}^m(p, \boldsymbol{\mu}) = C(\omega_0, \dot{p}) + G(R_0, p) - Y\hat{\theta} + \ddot{p}_r(t) - K_p s \tag{24}$$

where $\boldsymbol{\mu} = [\boldsymbol{\mu}_0 \quad \boldsymbol{\mu}_1 \quad \cdots \quad \boldsymbol{\mu}_{N-1}]^T$ is the actual unknown control vector.

B. Adaptive Attitude Tracking

In this section adaptive attitude tracking control laws will be presented that can be derived in a fashion similar to that done in the previous section for translational control. Each satellite in the formation is assumed to have three orthogonal reaction wheels; hence, the attitude control can be implemented in a decentralized fashion, as discussed earlier. In the attitude control as well, there are two uncertainties in the model: namely, the actual magnetic torque that acts on the satellite, due to the magnetic field of other satellites that can vary as much as 10%, depending upon whether the satellite is in the near- or far-field region, and the second uncertainty lies with the Earth's magnetic field, as discussed before. To account for these model uncertainties, an adaptive control scheme is proposed with the control and adaptation laws, respectively, as

$$\boldsymbol{\tau}_{\mathrm{RW}} = (\mathbf{H}_B + \mathbf{H}_{\mathrm{RW}}) \times \boldsymbol{\omega} + \hat{\boldsymbol{\tau}}^m + \hat{\boldsymbol{\tau}}^e + I\dot{\omega}_r - Y_\tau \hat{\theta}_\tau + K_\tau s_\tau \quad (25)$$

$$\dot{\hat{\theta}}_\tau = -\Gamma_\tau Y_\tau^T s_\tau \quad (26)$$

where $\hat{\boldsymbol{\tau}}^m$ and $\hat{\boldsymbol{\tau}}^e$ are the estimated torques on the satellite due to other satellite dipoles and the Earth's magnetic field, respectively. These can be computed using Eqs. (7) and (10). Y_τ is the gain matrix for the parameters $\hat{\theta}_\tau$ defined as

$$Y_\tau = \begin{bmatrix} \hat{\tau}_x^m & 0 & 0 & 0 & -\mu_z & \mu_y \\ 0 & \hat{\tau}_y^m & 0 & \mu_z & 0 & -\mu_x \\ 0 & 0 & \hat{\tau}_z^m & -\mu_y & \mu_x & 0 \end{bmatrix}$$

$\boldsymbol{\mu} = [\mu_x \quad \mu_y \quad \mu_z]^T$ is the dipole control vector for the satellite, $\hat{\theta}_\tau = [\gamma_{\tau x} \quad \gamma_{\tau y} \quad \gamma_{\tau z} \quad \tilde{B}_{ex} \quad \tilde{B}_{ey} \quad \tilde{B}_{ez}]^T$, K_τ is a positive definite diagonal gain matrix for the composite attitude tracking error defined as $s_\tau(t) = \tilde{\omega}(t) + \lambda_\tau \tilde{\alpha}(t)$, with $\tilde{\omega} = \omega - \omega_d$, $\tilde{\alpha} = \alpha - \alpha_d$, $\lambda_\tau > 0$ is a diagonal gain matrix, and $\dot{\omega}_r = \dot{\omega}_d - \lambda_\tau \dot{\alpha} + \lambda_\tau \dot{\alpha}_d$.

By using a procedure similar to that outlined in the previous section for the translational control, the attitude control law and the adaptation law can be shown to asymptotically follow a predefined, sufficiently smooth, attitude trajectory specified by α_d, ω_d, and $\dot{\omega}_d$. In this formulation attitude orientation vector α can be defined by using quaternion rotation axis components: i.e., $\alpha = [q_1 \quad q_2 \quad q_3]^T$. In the case of quaternions, the error in the orientation, i.e., $\tilde{\alpha}$, needs to be described by two successive quaternion rotations, since the error quaternion will be the current satellite attitude quaternion with respect to the commanded attitude quaternion [14]. Therefore, the difference operator on the quaternions to generate the error vector can be shown to be

$$\tilde{\alpha} = q' - q'_d = \begin{bmatrix} -q_1 & q_0 & q_3 & -q_2 \\ -q_2 & -q_3 & q_0 & q_1 \\ -q_3 & q_2 & -q_1 & q_0 \end{bmatrix} q_d \quad (27)$$

where prime indicates the last three components of the quaternion, i.e., $q' = [q_1 \quad q_2 \quad q_3]^T$, and q_d is the desired attitude quaternion.

C. Control Allocation Mechanism

A contribution of this paper is the description of an actuation allocation mechanism that allows achieving the commanded control forces and torques.

Note that vector Eq. (24) represents $3N - 3$ scalar equations for an N-satellite formation, whereas the control vector μ has length $3N$. Thus, this is effectively an under constrained system; i.e., we have more control variables than controllable degrees of freedom. This means that one of the dipoles, called free dipole, can be chosen arbitrarily and provides extra degree of freedom to control other parameters in the system (e.g., angular momentum, as discussed later in this section). Moreover, Eq. (2) shows that the dipole equations [left-hand side of Eq. (24)] are polynomials in the unknown vector $\boldsymbol{\mu}$. Since polynomial roots (in this case $\boldsymbol{\mu}$) have smooth dependence on parameters and polynomials have multiple roots, depending upon their degree, there are multiple solutions to Eq. (24) giving extra freedom in controlling other system parameters such as angular momentum.

As discussed in the above paragraph, one way of dynamic inversion to compute the control values is to fix one of the dipoles and solve the set of polynomial Eqs. (24). This way we have potentially infinite solutions (one for each value of the fixed dipole) and out of those infinite solutions the most desirable solution is the solution that minimizes the angular momentum buildup in the reaction wheels. This can be accomplished by posing the dynamic inversion problem as an optimization problem given below:

$$\boldsymbol{\mu} = \arg\min_{\boldsymbol{\mu}} \sum_{i=0}^{N-1} (\|\boldsymbol{\tau}_i^m + \boldsymbol{\tau}_i^e\| + \mu_i^{\mathrm{T}} W \mu_i) \quad \text{subject to Eq.(24)} \quad (28)$$

where W is a diagonal weighting matrix that helps in distributing the dipole strengths according to the coil sizes on each satellite. Note that optimization problem defined in Eq. (28) is essentially equivalent to adding an extra constraint to the control equation (24) in order to be able to generate a unique solution to the problem.

V. Angular Momentum Management Through Polarity Switching

As discussed earlier, angular momentum buildup in each satellite in the formation can be an issue with EMFF.

For near-Earth operation, however, disturbances due to gravity differentials create force pairs that can induce constant increase of the angular momentum of the system and disturbance torques due to interaction of Earth's magnetic field with the coils on each satellite.

As discussed in Sec. IV.C, in an N-satellite formation the desired forces can be implemented by setting up $N - 1$ dipoles. This leaves a free-dipole available, which can be used to solve the dipole equations in such as way so as to minimize the buildup of angular momentum in each satellite, as described in Sec. IV.B. However, as the Earth's magnetic field interacts with the coils on each vehicle, it produces permanent torques. Such torques are counteracted by the reaction wheels but the amount of torque that a reaction wheel can store is limited and is an important consideration in the power design of the attitude control system. The reaction wheels on each satellite can quickly become saturated even after adjusting the dipole solution for minimizing angular momentum buildup.

The important consideration for applying the Angular Momentum Management strategy herein proposed is based on a unique non-linearity of the magnetic torques. The force acting between a pair of dipoles depends upon the product of the individual dipole values. The torque caused by external slowly varying magnetic fields is inverted by inverting the polarity of the dipole created by a coil. On the other hand, if all coil-generated dipoles change their sign (i.e., flip all satellites dipoles by 180°), the force acting due to coil interactions does not change. By switching the polarity of all dipoles in the formation, the torque acting on the satellites due to Earth's magnetic field changes sign, but the torques and forces due to the other satellites in the system do not.

As can be seen from Eq. (10), it results in a net cancellation of the effect of the Earth's magnetic field on the average sense.

Practically speaking, this switching action will take finite time, depending upon the size of coils and power considerations; nevertheless, it can always be accomplished in a relatively small time as compared to the time constants of the formation error rates and changes in the Earth's magnetic field as the satellites translate over an orbit, as shown by the simulation results in the next section.

In a more sophisticated implementation, which is out of the scope of this paper, an angular momentum management (AMM) strategy

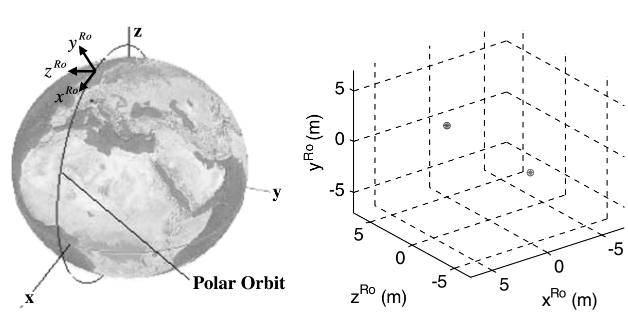

Fig. 2 Orbits of the formation (left) and position of satellites in orbital frame (right).

Table 1 Parameter values used in the simulation

Parameter	Value	Parameter	Value
Cross-track distance, m	10	K_τ, Ns	100
Leader mean motion ω_0, rad/s	1.1068×10^{-3}	$\lambda_\tau(1/\text{s})$	1
Mass of each satellite, kg	250	Γ_{fm} (gain for γ_i)	1
I, kg-m^2, same for all axes	20	Γ_{fd} (gain for d_i)	0.1
K_p, Nm^{-1} s	0.5	$\Gamma_{\tau m}$ (gain for τ_x)	1e − 3
λ, 1/s	0.02	$\Gamma_{\tau d}$ (gain for $\tilde{B}_e$)	1e − 7
Unknown Earth's B x component, T	1.0×10^{-5}	Relative dist. force F_{dx}, N	$1 \times 10^{-3} \sin(\omega_0 t)$
Unknown Earth's B y component, T	-0.5×10^{-5}	Relative dist. force F_{dy}, N	0.0
Unknown Earth's B z component, T	-2.0×10^{-5}	Relative dist. force F_{dz}, N	0.0

can be implemented at a lower level by monitoring the saturation levels of the reaction wheels on each satellite and switching the sign of the dipoles. For the simulation results presented in the next section, the AMM strategy was implemented in a periodically switching fashion and the results suggest the suitability of this simpler approach.

VI. Simulation Results

We will present simulation results for a two-satellite formation and a five-satellite formation in a circular polar orbit around Earth at an altitude of 500 km (see Fig. 2). In both cases the orbits of the individual satellites are non-Keplerian and indeed require forces to maintain their respective orbits.

The parameters used in the simulations are listed in Table 1. The dynamics include the second harmonic of the gravitational field as well as the Earth's magnetic field. Moreover, in the simulation a constant external magnetic field component (assumed unknown and not considered in the model) having strength equal to $[1 \times 10^{-5} - 0.5 \times 10^{-5} - 2.0 \times 10^{-5}]^T$ Wb/m^2 was also assumed in the simulation. Similarly, a relative disturbance force of 1 mN varying with the orbit was also used in the simulation.

The robustness of the approach is demonstrated with the results of the simulations, considering the disturbance forces included in the

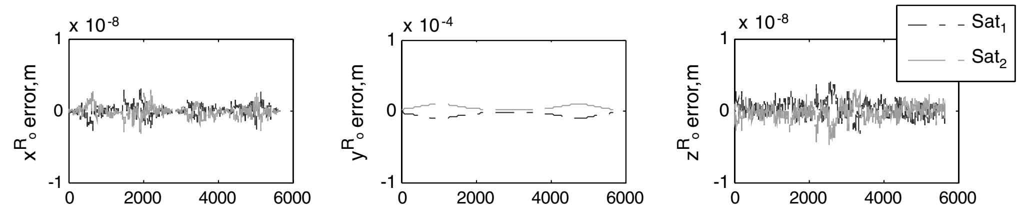

Fig. 3 Error in position in the orbit frame Ro.

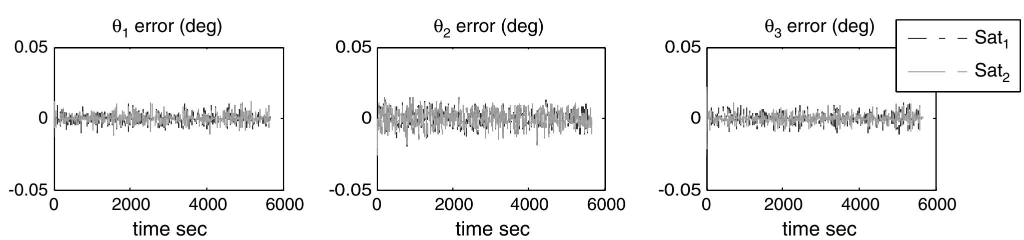

Fig. 4 Error in attitude (Euler angles).

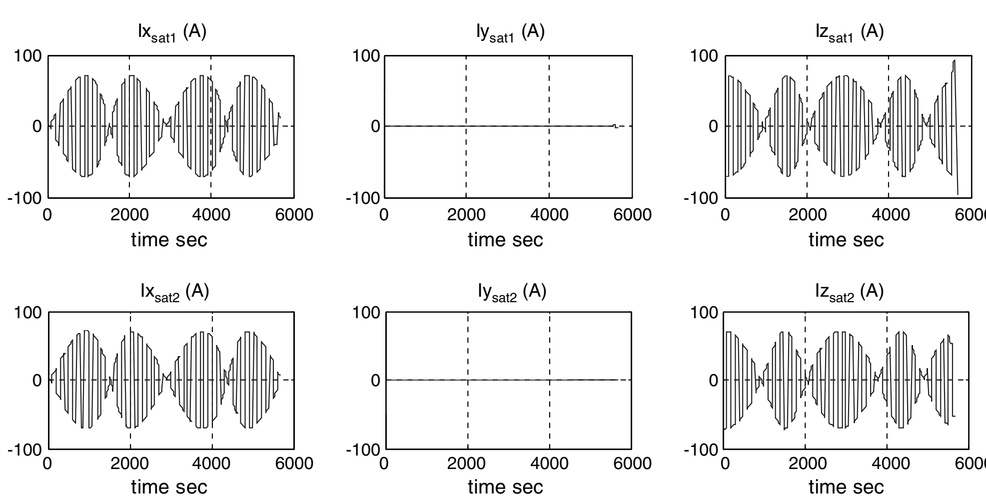

Fig. 5 Coil current inputs for satellite 1 (top) and satellite 2 (bottom).

simulation. More recent theoretical results can provide robustness guaranties for specific cases and should be part of future research in the control approach for EMFF systems.

A. Example 1

The first example considers a two-satellite formation motivated by the applications in synthetic aperture radar technology, where the cross-track distance between the two satellites can act as the baseline of the interferometer and can yield information about ground elevation differences [15].

For this application the two satellites are to be kept at a fixed distance from each other in cross-track position, as shown in Fig. 2. Indeed, as mentioned before, a continuous force along the cross-track axis is required to maintain their non-Keplerian orbits. In this simulation a switching time of 50 s was used for AMM. Figures 3 and 4 give the position error and attitude error of the follower satellite for one orbital period. Figure 5 shows the follower satellite coil currents

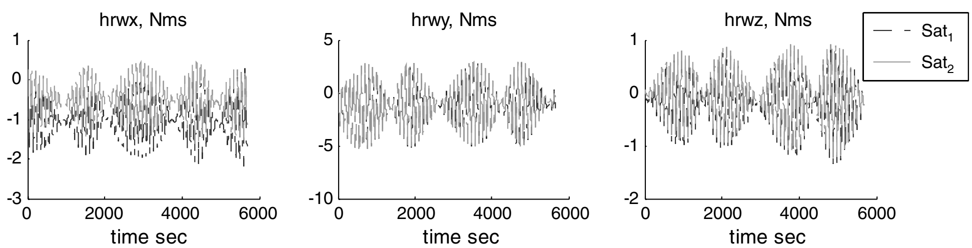

Fig. 6 Reaction wheel angular momentum.

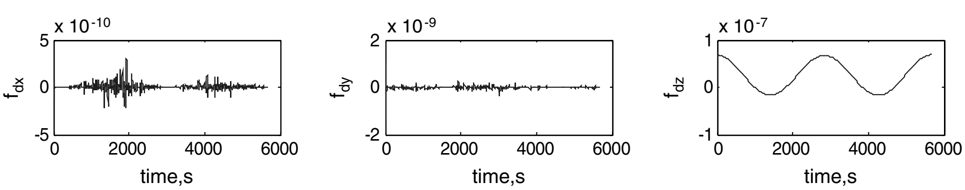

Fig. 7 Adaptive gains for relative disturbance force.

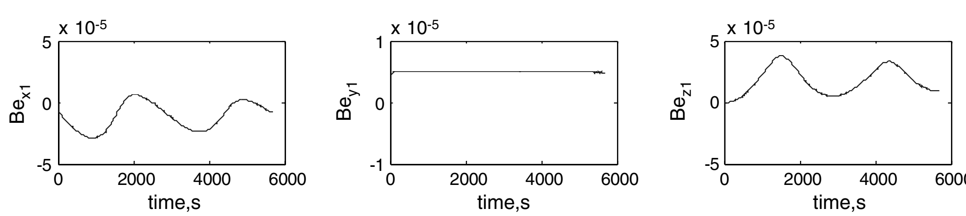

Fig. 8 Adaptive gains for unknown component of the Earth's magnetic field (leader satellite).

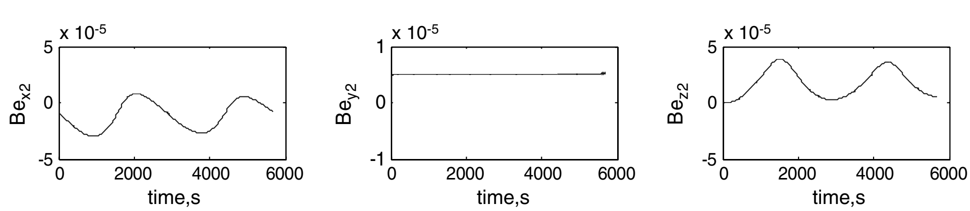

Fig. 9 Adaptive gains for unknown component of the Earth's magnetic field (follower satellite).

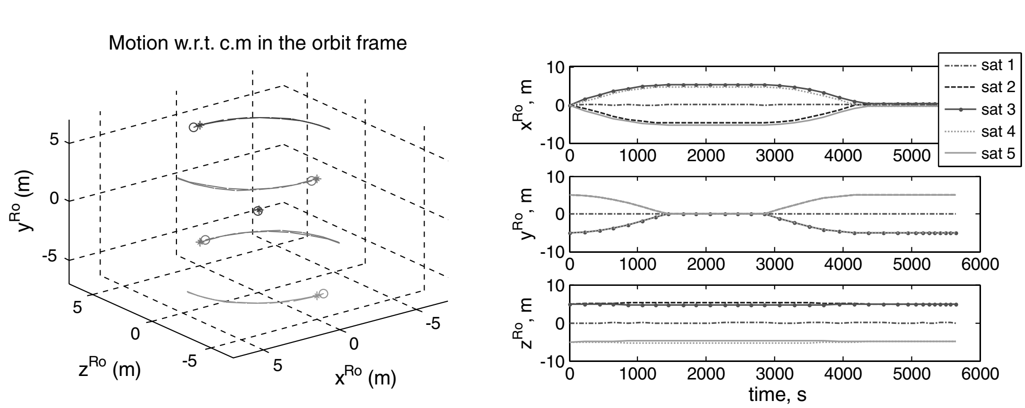

Fig. 10 Trajectories in the orbital frame for the five-satellite formation.

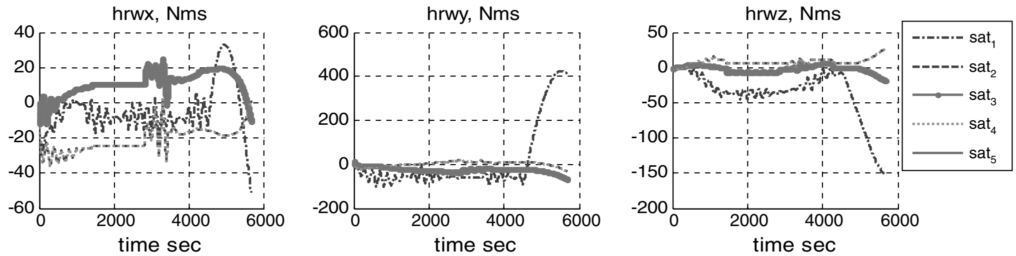

Fig. 11 Reaction wheel angular momentum for satellites in five-satellite formation.

required to hold the formation and switching action is evident from this figure. It should be noted that for the specified parameters of the simulation, the solution of the optimization problem in Eq. (28) converges to solutions that use the transversal coils (x, z) instead of the aligned (y) coils, minimizing the angular momentum into the system. Figure 6 gives the reaction wheel angular momentum buildup of the follower satellite, and it is clear from this figure that the angular momentum buildup was limited by the switching action; otherwise, angular momentum would build up continuously, saturating the reaction wheels quickly.

Figures 7–9 show the values of different adaptation parameters. The near-field adjustment parameters are not given, since for this simulation, the satellites are in the far-field region and these parameters are zero. As can be seen from these figures, the adaptation

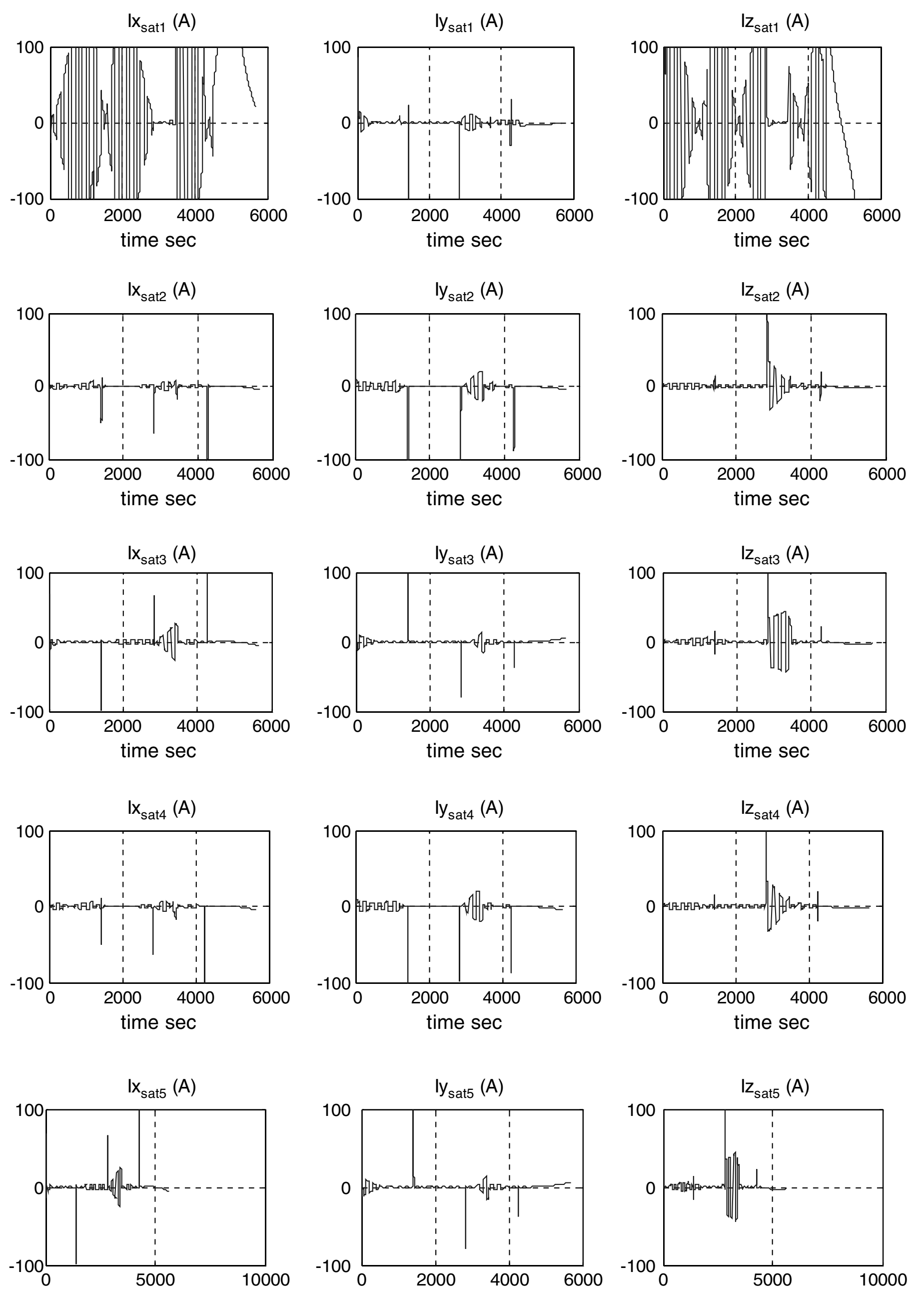

Fig. 12 Coil currents for satellites 1–5, five-satellite formation.

laws effectively vary the estimated values of the parameters in order to adjust for unknown external disturbance forces and torques.

B. Example 2

In a second example, a five-satellite formation maintains a square formation with one of the satellites in its center while performing a plane rotation maneuver along the y^{R_o} axis. The formation performs a rotation about its local axis, maintains that orientation for one-fourth of a period, and rotates back to reach its initial position. The motivations behind this type of formation are proposed missions such as the interferometric millimeter-wave imaging application [16].

Figure 10 shows the positions of the formation in the F^{R0} frame for a time of one revolution around the Earth. Figure 11 shows the magnitude of the angular momentum for all the satellites. Figure 12 shows the currents in the coils for each of the vehicles and each one of their body-fixed axes.

For this simulation, a switching time for AMM of 100 s was used during the first seven-eighths of a period (4608 s). To demonstrate the importance of implementing this strategy, the AMM is turned off after 4608 s. Notable from plots in Fig. 11 is the uncontrolled angular momentum buildup as the dipoles are selected only to achieve the desired control forces and torques without considering the continuous disturbance torques.

VII. Conclusions

This paper presented an approach to control of a formation of vehicles in LEO orbit using electromagnetic forces. Despite the fact that Earth's magnetic field acts as a strong disturbance source in electromagnetic formation-flying systems, the inherent nonlinearity of the magnetic dipoles can be used to mitigate its effects by applying a switching action. An adaptive control approach is presented that is intended to compensate for the effect of unknown parameters, especially on the Earth's magnetic field models. Simulations show the feasibility of a control allocation method based on the solution of a quadratic optimization problem as well as the suitability of adaptive control techniques for low-Earth-orbit applications. Future work includes preparation of experiments for verifying the models and the control laws on a two-dimensional ground testbed.

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